Supplemental Material

Strictly connected to any definition of modularity are the ideas (a) that the operations of different modules (i.e. controllers) may be combined to generate a repertoire or results and (b) that the different modules may act independently without interfering with one another.

Given these two requirements, consider two hypothetical modules A and B, and suppose the activation of A corresponds to a motor response F_A and the activation of B to motor response F_B. In addition, suppose that the simultaneous activation of A and B produces a combination of the two responses $F_{AB} = \Gamma(F_A, F_B)$, where Γ denotes a smooth functional. It is quite straightforward to see that among all possible rules of combination, the principle of vector summation $f(F_A, F_B) = F_A + F_B$ is the one that is most consistent with the two requirements of modularity, (a) and (b). The rule of vector summation is equivalent to imposing the following properties on the combination functional $\Gamma(\cdot)$

First, that the two modules play a symmetric role in the combination. With some abuse of notation, one can express this as

$$\frac{\partial \Gamma}{\partial F_A} = \frac{\partial \Gamma}{\partial F_B} \tag{1}$$

From this condition it follows that the combination of these modules must be a function of the sum of the two responses $\Gamma(F_A, F_B) = \Gamma(F_A + F_B)$.

Second, the output generated by each module, when it acts in isolation, is the same as when it acts in combination.

$$\left\{ \begin{array}{c} F_A = \Gamma(F_A, 0) \\ F_B = \Gamma(0, F_B) \end{array} \right\} \tag{2}$$

This is a boundary condition that constrains the solution to plain summation $\Gamma(F_A + F_B) = F_A + F_B$. From these "rules" of modularity we have established that linear combination properties are not merely a matter of computational convenience. In a deeper sense, vector summation corresponds to optimal modularity as established by the conditions (2) and (3). These two conditions on Γ are both necessary and sufficient for vector summation.In this respect we may state that vector summation is the most fundamental rule of combination when the goal is to obtain modularity.